Integral Bee 2026

MathSoc x TPSA

Qualifiers

Round 1

Round 1

$$\int \tan(\theta)\,\mathrm{d}\theta$$

$$\int_{\mathbb{R}} \frac{\mathrm{d}x}{1+e^{|x|}}$$

$$\int_0^{1/2} \frac{x}{(1-x)^2}\,\mathrm{d}x$$

$$\int e^x \sin(x)\,\mathrm{d}x$$

$$\int \mathrm{rain}\,\mathrm{d}\,\mathrm{rain}$$

Round 1 - Solutions

$$\int \tan(\theta)\,\mathrm{d}\theta = -\ln(\cos(x))$$

$$\int_{\mathbb{R}} \frac{\mathrm{d}x}{1+e^{|x|}} = \ln(4)$$

$$\int_0^{1/2} \frac{x}{(1-x)^2}\,\mathrm{d}x = 1 - \ln(2)$$

$$\int e^x \sin(x)\,\mathrm{d}x = \frac{e^x}{2}(\sin(x) - \cos(x))$$

$$\int \mathrm{rain}\,\mathrm{d}\,\mathrm{rain} = \frac{1}{2}(\mathrm{rain})^2$$

Qualifiers
Round 2

Round 2

$$\int_0^{\frac{\pi}{4}} \frac{dx}{\sqrt{1-x^2}}$$

$$\int_0^{\infty} x^{67} e^{-x}\, dx$$

$$\int \ln x\, dx$$

$$\int \frac{\tan x}{\cos^2 x}\, dx$$

$$\int_0^{\frac{\pi}{2}} \sin^7(x)\cos(x)\, dx$$

Round 2 - Solutions

$$\int_0^{\frac{\pi}{4}} \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin\left(\frac{\pi}{4}\right)$$

$$\int_0^{\infty} x^{67} e^{-x}\,\mathrm{d}x = 67!$$

$$\int \ln x\,\mathrm{d}x = x\ln x - x + C$$

$$\int \frac{\tan x}{\cos^2 x}\,\mathrm{d}x = \frac{\tan^2 x}{2} + C$$

$$\int_0^{\frac{\pi}{2}} \sin^7(x)\cos(x)\,\mathrm{d}x = \frac{1}{8}$$

Qualifiers
Round 3

Round 3

$$\int \frac{1}{\sqrt{1+2e^x+e^{2x}}}$$

$$\int_{-a}^{a} \sqrt{a^2-x^2}\,\mathrm{d}x$$

$$\int_{0}^{\infty} \frac{\ln(x)}{1+x^2}\,\mathrm{d}x$$

$$\int_{0}^{2026} \frac{\mathrm{d}x}{1+\{x\}^2}$$

$$\int_{0}^{1013} \lceil \log_{\frac{1}{2}}\{x\} \rceil\,\mathrm{d}x\,,\ \text{where } \{x\} = x - \lfloor x \rfloor$$

Round 3 - Solutions

$$\int \frac{1}{\sqrt{1+2e^x+e^{2x}}} = \ln(2)$$

$$\int_{-a}^{a} \sqrt{a^2-x^2}\,\mathrm{d}x = \frac{\pi a^2}{2}$$

$$\int_{0}^{\infty} \frac{\ln(x)}{1+x^2}\,\mathrm{d}x = 0$$

$$\int_{0}^{2026} \frac{\mathrm{d}x}{1+\{x\}^2} = \frac{1013\pi}{2}$$

$$\int_{0}^{1013} \left\lceil \log_{\frac{1}{2}}\{x\} \right\rceil \mathrm{d}x = 2026$$

Quarter Finals

Leg 1

Round 1

$$\int_{x^2+y^2=1} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

Round 1 - Solutions

$$\int_{x^2+y^2=1} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = 2\pi$$

Round 2

$$\int \frac{1 + \ln(x)}{x^{-x}} \, \mathrm{d}x$$

Round 2 - Solutions

$$\int \frac{1 + \ln(x)}{x^{-x}} \, \mathrm{d}x = x^x$$

Round 3

$$\int_0^{\frac{\pi}{2}} \arctan(x) + \arctan\left(\frac{1}{x}\right) \mathrm{d}x$$

Round 3 - Solutions

$$\int_0^{\frac{\pi}{2}} \arctan(x) + \arctan\left(\frac{1}{x}\right) \mathrm{d}x = \frac{\pi^2}{4}$$

Quarter Finals

Leg 2

Round 1

$$\int \frac{1 + \cot(x)}{1 - \cot(x)} \, dx$$

Round 1 - Solutions

$$\int \frac{1+\cot(x)}{1-\cot(x)}\,\mathrm{d}x = \ln(\cos(x)-\sin(x))$$

Round 2

$$\int \frac{\mathrm{d}x}{\sinh(x)}$$

Round 2 - Solutions

$$\int \frac{\mathrm{d}x}{\sinh(x)} = \ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Round 3

$$\int \frac{4x}{x^4 + 4} dx$$

Round 3 - Solutions

$$\int \frac{4x}{x^4+4}\,dx = \arctan(x-1) - \arctan(x+1) + C$$

$$= \arctan\left(\frac{x^2}{2}\right) + C$$

Quarter Finals

Leg 3

Round 1

$$\int \frac{1}{(1-\beta^2)^{3/2}}\,\mathrm{d}\beta$$

Round 1 - Solutions

$$\int \frac{1}{(1-\beta^2)^{3/2}}\,\mathrm{d}\beta = \frac{\beta}{\sqrt{1-\beta^2}}$$

Round 2

$$\int_0^{100} \lceil \sqrt{x} \rceil \, dx$$

Round 2 - Solutions

$$\int_0^{100} \lceil \sqrt{x} \rceil = 715$$

Round 3

$$\int_{-\infty}^{\frac{1}{3}\ln(2)} \sqrt{36e^{6x} - 9e^{12x}}\,\mathrm{d}x$$

Round 3 - Solutions

$$\int_{-\infty}^{\frac{1}{3}\ln(2)} \sqrt{36e^{6x} - 9e^{12x}}\,\mathrm{d}x = \pi$$

Quarter Finals

Leg 4

Round 1

$$\int_{\mathbb{R}} \frac{1}{x^2 + y^2} \, \mathrm{d}y$$

Round 1 - Solutions

$$\int_{\mathbb{R}} \frac{1}{x^2 + y^2} \, \mathrm{d}y = \frac{\pi}{|x|}$$

Round 2

$$\int_0^1 \frac{36x^2 + 8x - 15}{12x^3 + 4x^2 - 15x - 7} \, \mathrm{d}x$$

Round 2 - Solutions

$$\int_0^1 \frac{36x^2 + 8x - 15}{12x^3 + 4x^2 - 15x - 7}\,\mathrm{d}x = \ln\left(\frac{6}{7}\right)$$

Round 3

$$\int_0^\infty \frac{e^{x+2}}{e^{e^x}\sqrt{e^x-1}}\,\mathrm{d}x$$

Rouxnd 3 - Solutions

$$\int_0^\infty \frac{e^{x+2}}{e^{e^x}\sqrt{e^x-1}}\,\mathrm{d}x = e\sqrt{\pi}$$

Semi Finals

Leg 1

Round 1

$$\int_1^e \frac{\ln\left(2x - \frac{x^2}{2x - \frac{x^2}{\cdots}}\right)}{2x - \frac{x^2}{2x - \frac{x^2}{\cdots}}}\,\mathrm{d}x$$

Round 1 - Solutions

$$\int_1^e \frac{\ln\left(2x - \frac{x^2}{2x - \frac{x^2}{\cdots}}\right)}{2x - \frac{x^2}{2x - \frac{x^2}{\cdots}}}\,\mathrm{d}x = \frac{1}{2}$$

Round 2

$$\int_0^1 e^{x^2} - 1 dx + \int_0^{e-1} \sqrt{\ln(x+1)}\, dx$$

Round 2 - Solutions

$$\int_0^1 e^{x^2} - 1 dx + \int_0^{e-1} \sqrt{\ln(x+1)}\, dx = e - 1$$

Round 3

Let 🐝 $\in \mathbb{R}^d$ be a vector in an inner product space

$$\int_{\mathbb{R}^d} \mathrm{d}^d🐝 \, e^{-\langle 🐝, 🐝 \rangle}$$

Round 3 - Solutions

$$\int_{\mathbb{R}^{\mathsf{d}}} \mathrm{d}^{\mathsf{d}}\text{🐝}\, \mathsf{e}^{-\langle \text{🐝}, \text{🐝} \rangle} = (\sqrt{\pi})^{\mathsf{d}}$$

Semi Finals

Leg 2

Round 1

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{(x^2 + \frac{1}{x^2} - 1)^3}$$

Round 1 - Solutions

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}x}{(x^2 + \frac{1}{x^2} - 1)^3} = \frac{3\pi}{8}$$

Round 2

$$\int_{\pi/4}^{3\pi/4} \delta(\tan(x))\,dx$$

Round 2 - Solutions

$$\int_{\pi/4}^{3\pi/4} \delta(\tan(x))dx = 0$$

Round 3

$$\int \sqrt{\frac{x}{1+x^3}}\,\mathrm{d}x$$

Round 3 - Solutions

$$\int \sqrt{\frac{x}{1+x^3}}\,\mathrm{d}x = \frac{2}{3}\operatorname{arcsinh}(x^{3/2})$$

Grand Finals 

Round 1

$$\sum_{n=0}^{\infty} \int_{0}^{1} x^n \ln(x)\, dx$$

Round 1 - Solutions

$$\sum_{n=0}^{\infty} \int_0^1 x^n \ln(x)\,dx = -\frac{\pi^2}{6}$$

Round 2

$$\int_{i\sqrt{\frac{\pi}{3}}}^{i\sqrt{\frac{\pi}{6}}} \frac{2x\sqrt{3}+2x\tan\left(x^2\right)}{(1-\sqrt{3}\tan(x^2))(\cos^2(x^2+\frac{\pi}{3}))}\,\mathrm{d}x$$

Round 2 - Solutions

$$\int_{i\sqrt{\frac{\pi}{3}}}^{i\sqrt{\frac{\pi}{6}}} \frac{2x\sqrt{3}+2x\tan\left(x^2\right)}{(1-\sqrt{3}\tan(x^2))(\cos^2(x^2+\frac{\pi}{3}))}\,\mathrm{d}x = \frac{1}{6}$$

Round 3

$$\int_1^{\pi} x\sqrt{x\sqrt[3]{x\sqrt[4]{x\cdots}}}$$

Round 3 - Solutions

$$\int_1^{\pi} x\sqrt{x\sqrt[3]{x\sqrt[4]{x\cdots}}} = \frac{\pi^e - 1}{e}$$

Round 4

$$\int \frac{\sqrt{e^{\sin x}}(\cos x \ln(\cos x) + \tan x)}{2\left(\ln(\cos x)\right)^{\frac{3}{2}}} dx$$

Round 4 - Solutions

$$\int \frac{\sqrt{e^{\sin x}}(\cos x \ln(\cos x) + \tan x)}{2\left(\ln(\cos x)\right)^{\frac{3}{2}}} dx = \sqrt{\frac{e^{\sin x}}{\ln(\cos x)}}$$

Round 5

$$\int_0^1 \mathrm{d}x\, x^m \frac{x}{1 + x\ln(x)}, \quad \text{Series for } |x\ln(x)| < 1.$$

Round 5 - Solutions

$$\int_0^1 \mathrm{d}x\, x^m \frac{x}{1+x\ln(x)} = \sum_{n=0} \left(\frac{1}{n+m+2}\right)^{n+1} \Gamma(n+1)$$